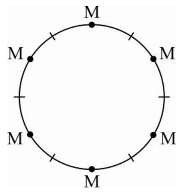
Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 1	DAILY TUTORIAL SHEET 5

101.(B) Consider the host and the two particular persons as a single unit. So, there are effectively 19 persons. These 19 persons can be arranged in $\lfloor 18$ ways and there are $\lfloor 2$ ways of arranging the two persons on either side of the host.

102.(D) Answer = $\dfrac{\lfloor 9}{2}$

103.(A) Since the seats are numbered, there is no circular symmetry. The problem is simply arranging people in a line i.e. ${}^nC_m m!$



104.(C) Total number of ways = (11 − 1)! = 10!

105.(A) First, we fix the position of men, the number of ways to sit men = 5!

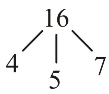
and the number of ways to sit women = ${}^6P_5$.

$\therefore$ Total number of ways = $5! \times {}^6P_5 = 5! \times 6!$

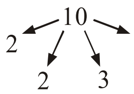
106.(A) $A - x, B - x + 1, C - x + 3$

$x + x + 1 + x + 3 = 16 \quad \Rightarrow \quad x = 4$

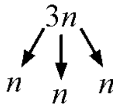
$A - 4, B - 5, C - 7$



No. of solutions $= \dfrac{16!}{4!5!7!}$

107.(D) 

Number of ways $= \dfrac{10!}{2!\,2!\,3!\,3!} = 25200$

108.(B) 

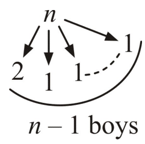
$\dfrac{(3n)!}{\underbrace{n!n!n!}_{\text{arrangement}}} = k$

Number of ways to divide $= \dfrac{(3n)!}{n!\,n!\,n!} \times \dfrac{1}{3!}$; Hence, number of ways to divide $= \dfrac{k}{3!}$

109.(B) Select 1 out of n boys for zero toys in ${}^nC_1$ ways. Now n toys to be distributed among $n-1$.

Boys [obviously, now 1 will get two and others will get one each]

Number of ways $= \dfrac{n!}{2!\,1!\,1!....1!} \times \dfrac{1}{(n-2)!} \times (n-1)! = \dfrac{n!}{2} \times (n-1)$



Total number of ways $= {}^nC_2 \times n!$

110.(A) Take out Ace, King, Queen and Jack of four suits and stack them into four groups according to their suit. Distribute the remaining cards in $\dfrac{(36!)}{(9!)^4}\,ways$.

Now, permute those 4 groups in 4! ways.

111.(D) Using *PIE* $3^8 - {}^3C_1 \times 2^8 + {}^3C_2$ [PIE: Principle of inclusion and exclusion]

112.(A)

(25 paise)	4 in no.
(50 paise)	3 in no.
(1 Rupee)	2 in no.

Number of ways to give atleast one Rupee = total ways − (Number of ways to give less than 1 Rupee)

Total ways $= (4+1)(3+1)(2+1) = 60$

[Includes the case when no money is given]
No. of ways to give less than 1 Rupee can be found as:

25 - Paise	50 - Paise	1 - Rupee	
3	0	0	75 paise given
1	1	0	75 paise given
0	1	0	50 paise given
2	0	0	50 paise given
1	0	0	→ 25 paise given
0	0	0	→ No money is given

Required number of ways $= 60 - 6 = 54$

113.(A) A_1 is above A_2

Select 2 places for A_1 and A_2 in ${}^{10}C_2$ ways and arrange A_1 and A_2 in 1 way and others in 8! ways.

$${}^{10}C_2 \times 8! = \frac{10!}{8!\,2!} \times 1 \times 8! = \frac{10!}{2!}$$

Another approach: Total permutations = 10!
Total permutations with A_1 above A_2 = Total permutations with A_2 above A_1 ; Hence 10!/2

114.(C) $\overline{a_1}\,\overline{a_2}\,\overline{a_3}\,\overline{a_4}$

Select 4 digits out of 10 in ${}^{10}C_4$ ways.

As order is fixed for arrangements, $[\because\ a_1 > a_2 > a_3 > a_4]$

Hence No. of ways = (No. of ways to select) $\times 1 = \frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} = 210$

115.(B) We will follow the steps as shown:

Step I : Select three places for A, B and C to speak in ${}^{10}C_3$ ways.

Step II: Arrange A, B, C on selected seats in 1 way.

Step III: Arrange remaining 7 persons in 7! Ways.

Total no. of ways $= \left({}^{10}C_3 \times 1\right) \times 71 = \frac{10!}{3!} = \frac{10!}{6}$

116.(B) Group of people $\equiv$ no. of different forecasts that can be done.

3 kinds of forecasts $\equiv$ win, loss, or draw

Number of ways $= 3 \times 3 \times 3 \times 3 \times 3$; Hence $n = 243$

117.(B) $\underset{4}{(1,}\ \underset{3}{2,}\ \underset{2}{3,}\ \underset{1}{4)}\ \underset{1}{5}\ \underset{4}{(6,}\ \underset{3}{7,}\ \underset{2}{8,}\ \underset{1}{9)}$

Hence no. of ways $4! \times 4! = (4!)^2$ = ways

118.(B) Total ways = ${}^{n}C_3$

Total ways in which a particular child is included = ${}^{n-1}C_2$

${}^{n}C_3 = {}^{n-1}C_2 + 84 \quad \Rightarrow \quad \frac{n(n-1)(n-2)}{6} = \frac{(n-1)(n-2)}{2} + 84$

$\Rightarrow \quad (n-1)(n-2)(n-3) = 84 \times 6 = 9 \times 8 \times 7 \quad \Rightarrow \quad n = 10$

119.(D) $A = \{a_1, a_2, a_3, \ldots, a_n\}$

Four possible cases can exist

Case I : $a_i \in P, a_1 \in Q$

Case II : $a_i \in P, a_i \notin Q$

Case III : $a_i \notin P, a_i \in Q$

Case IV : $a_i \notin P, a_i \notin Q$

For $P \cap Q$ to contain exactly two elements, we follow the following steps:

Step I : Select 2 elements from n elements in ${}^{n}C_2$ ways

Step II : Favourable cases for selected two elements is case I.

Step III : Favourable cases for rest $n-2$ elements are case II, case III, Case IV.

Hence for $n-2$ elements, each has 3 options, or they can be arranged in 3^{n-2} ways.

No. of ways $= \left({}^{n}C_2 \times 1\right) \times 3^{n-2} = {}^{n}C_2 . 3^{n-2}$

120.(B) This can be done in four mutually exclusive ways as follows:

	Row R_1	**Row R_2**	**Row R_3**	**Number of ways**
I.	1	3	2	$\left({}^{2}C_1\right)\left({}^{4}C_3\right)\left({}^{2}C_2\right) = 8$
II.	1	4	1	$\left({}^{2}C_1\right)\left({}^{4}C_4\right)\left({}^{2}C_1\right) = 4$
III.	2	2	2	$\left({}^{2}C_2\right)\left({}^{4}C_2\right)\left({}^{2}C_2\right) = 6$
IV.	2	3	1	$\left({}^{2}C_2\right)\left({}^{4}C_3\right)\left({}^{2}C_1\right) = 8$
Total				26

121.(B) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^{2}C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in ${}^{5}C_2$ ways and they can be arranged in remaining seats in 2 ways.

Therefore, the required number of ways in which the car can be filled is $2 \times {}^{5}C_2 \times {}^{2}C_1 = 40$.

122.(A) They can be made to sit in $(2n)!\,(m!)(m-1)$ ways.

123.(D) Choose 3 seats is ${}^{10}C_3$ ways and arrange the remaining people in 7! ways.

124.(D) Sum of 4 natural numbers = 10 if numbers are $(1, 2, 3, 4)$ sum of two numbers = sum of remaining two

Case I: consider main diagonal first for (1 & 4)

1	
	4

2 and 3 can be arranged in 2! ways.

4	
	1

2 and 3 can be arranged in 2! ways.

Case II: Now, consider the off-diagonal for (1 & 4)

In similar way to main diagonal, it can be done in $(2! \times 2!)$ ways.

Total number of ways $= (2! \times 2!) + (2! \times 2!) = 2 \times (2! \times 2!)$

125.(B) Since a true/false type question can be answered in 2 ways either by marking it true or false. So, there are 2 ways of answering each of the 5 questions. So, total number of different sequences of answers $= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

Out of these 32 sequences of answers there is only one sequence of answering all the five questions correctly. But no student has written all the correct answers and different students have given different sequence of answers. So,

Maximum number of students in the class = Number of sequences except one sequences in which all answers are correct = $32 - 1 = 31$